

AN EARTHLY MODEL OF THE DIVINE COINCIDENCE

Pascal Michailat, Emmanuel Saez

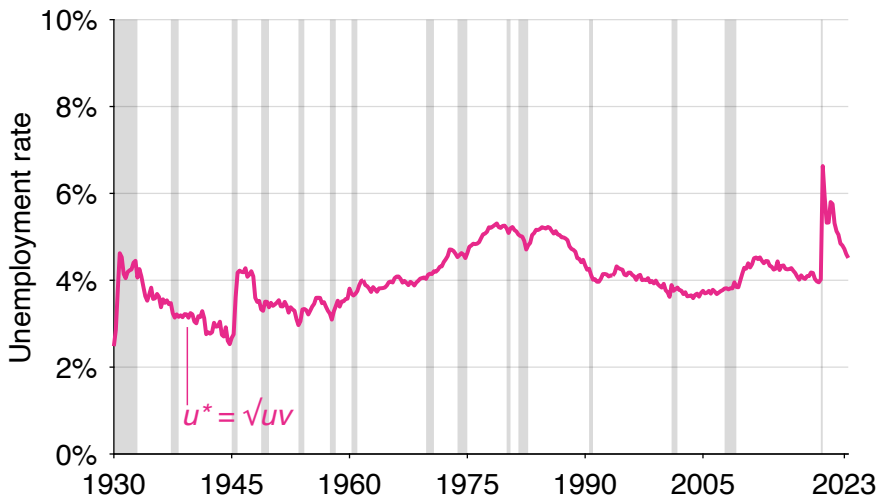
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Available at <https://pascalnichailat.org/15/>

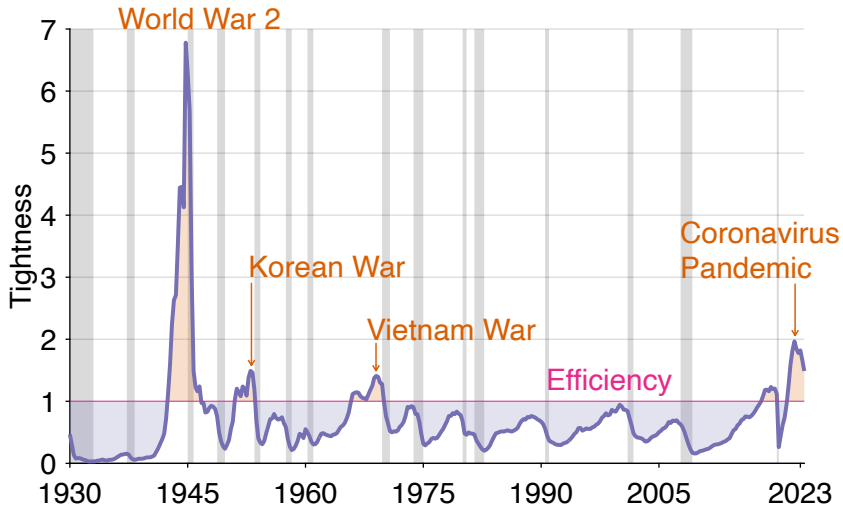
THE FED'S DUAL MANDATE

- responsibility of the Federal Reserve “to promote effectively the goals of **maximum employment, stable prices**”
 - Federal Reserve Reform Act of 1977
- stable prices: $\pi^* = 2\%$
 - Statement on Longer-Run Goals & Monetary Policy Strategy (2012)
- maximum employment: $u^* = \sqrt{uv}, \theta^* = 1$
 - proposal by Michaillat, Saez (2023)
 - u^*, θ^* maximize social welfare

$u^* = \sqrt{uv}$ AVERAGES 4.1% OVER 1930–2023



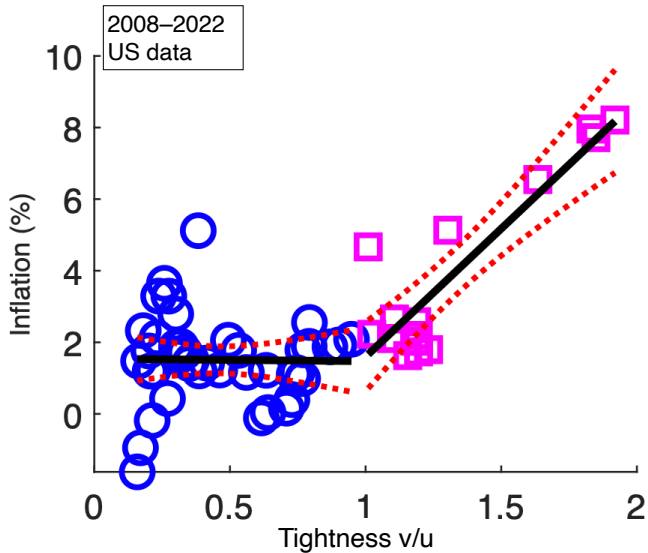
US LABOR MARKET IS GENERALLY INEFFICIENTLY SLACK



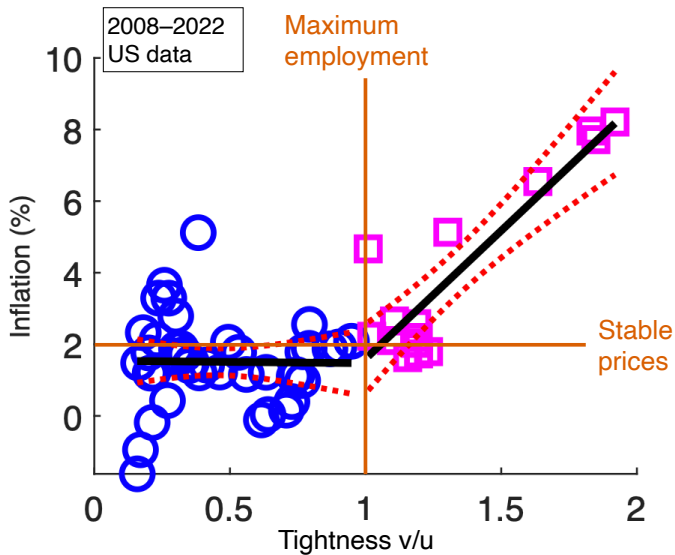
TRADITIONALLY, THE TWO MANDATES ARE NOT CONSISTENT

- under **traditional Phillips curve**: no guarantee that (u^*, π^*) is on curve
- under **accelerationist Phillips curve**: no guarantee that the NAIRU maximize social welfare
 - all other unemployment rate are inconsistent with stable inflation
- in **New Keynesian model** with unemployment fluctuations: wage rigidity breaks down divine coincidence
 - Blanchard, Gali (2010) see divine coincidence as unrealistic

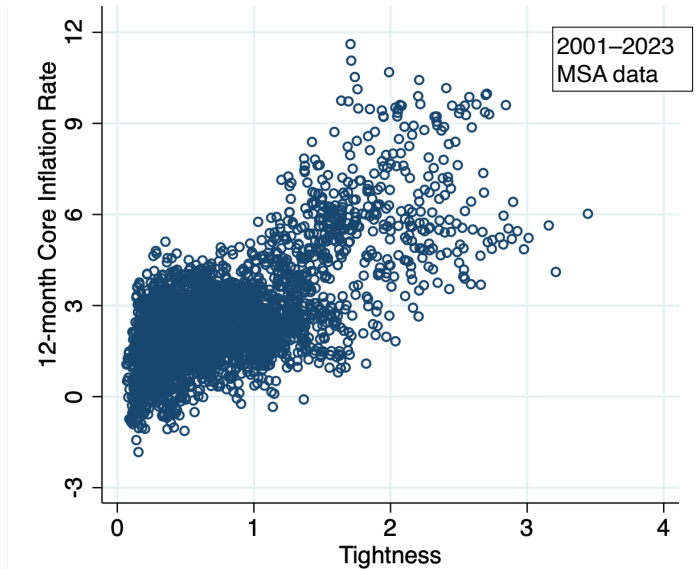
BUT: BENIGNO, EGGERTSSON (2023)



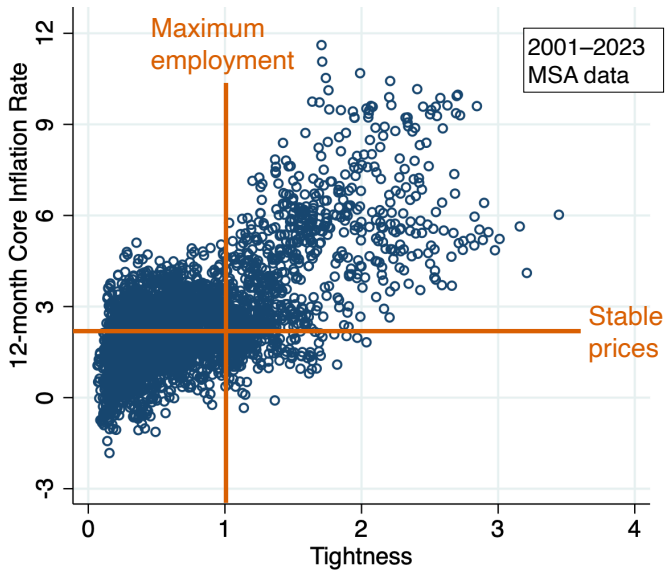
BUT: BENIGNO, EGGERTSSON (2023)



BUT: GITTI (2023)



BUT: GITTI (2023)



DIVINE COINCIDENCE APPEARS QUITE NATURALLY IN A MATCHING MODEL

- business-cycle model from Michaillat, Saez (2022)
 - sellers find customers through **matching** \Rightarrow unemployment
 - **utility from wealth** \Rightarrow nondegenerate aggregate demand
- price competition through **directed search** (Moen 1997)
- price rigidity from **quadratic price-adjustment costs** (Rotemberg 1982)
- \leadsto divine coincidence appears: $\pi = \pi^*$ iff $u = u^*$
- other properties of the model:
 - permanent zero-lower-bound episodes
 - fluctuations in unemployment & inflation
 - with kink in Phillips curve: fluctuations in unemployment in bad times but fluctuations in inflation in good times

DESCRIPTION OF THE MODEL

UNEMPLOYED WORKERS AND RECRUITERS

- people are organized in large households
- services are traded through long-term relationships
 - people are full-time employees in other households
 - employment relationships separate at rate $s > 0$
- household k has l_k workers producing services
 - y_{jk} workers work for household j
 - $y_k = \int_0^1 y_{jk}(t) dk$ workers are employed
 - $U_k = l_k - y_k$ **unemployed workers are at shop k**
- household j sends V_{jk} employees from household k to recruit workers at shop k
 - $V_k = \int_0^1 V_{jk}(t) dj$ **recruiters are at shop k**

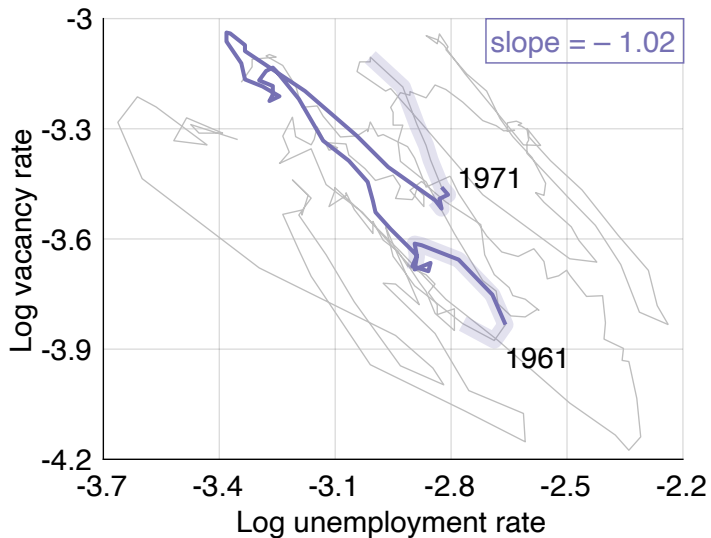
MATCHING BETWEEN WORKERS AND EMPLOYERS

- matching function determines flow of hires at shop k :

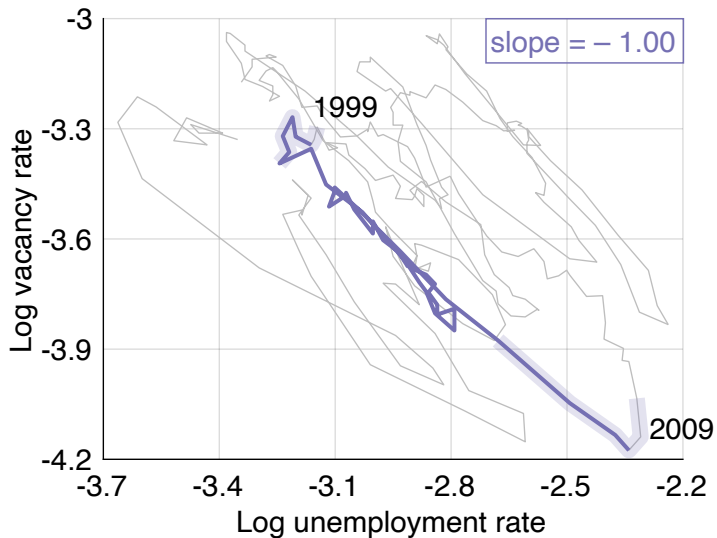
$$h_k = h(U_k, V_k) = \omega \cdot \sqrt{U_k \cdot V_k} - s \cdot U_k$$

- matching function has standard properties
 - constant returns to scale
 - $h = 0$ when $U = 0$
 - increasing in V and U (as long as unemployment $< 50\%$)
 - concave in V and U
- market tightness $\theta_k = V_k/U_k$ determines trading rates
 - job-finding rate: $f(\theta_k) = h_k/U_k = \omega \cdot \sqrt{\theta_k} - s$
 - recruiting rate: $q(\theta_k) = h_k/V_k = \omega/\sqrt{\theta_k} - s/\theta_k$

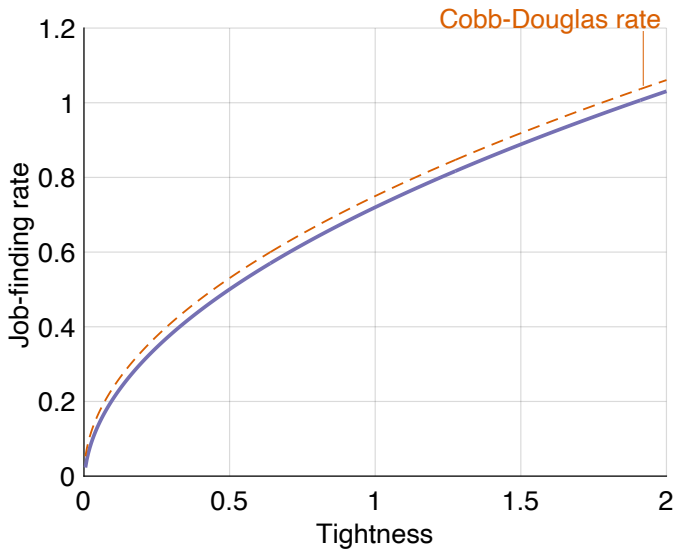
US BEVERIDGE CURVE \approx HYPERBOLA



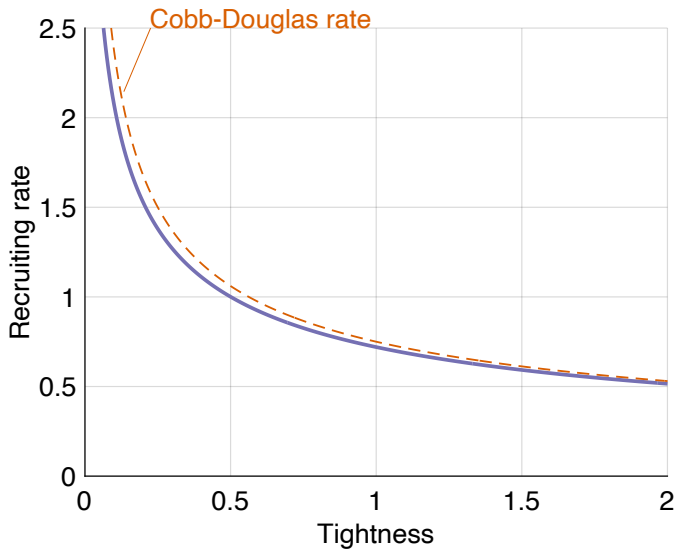
US BEVERIDGE CURVE \approx HYPERBOLA



MATCHING RATES BETWEEN WORKERS AND EMPLOYERS



MATCHING RATES BETWEEN WORKERS AND EMPLOYERS



BALANCED FLOWS AND UNEMPLOYMENT RATE

- number of employed workers in household k :

$$\dot{y}_k = f(\theta_k) \cdot U_k - s \cdot y_k = f(\theta_k) \cdot U_k - s \cdot [l_k - U_k]$$

- US labor-market flows are balanced (Michaillat, Saez 2021)
 - assume that flows are balanced in all (j, k) cells
 - in particular flows are balanced in household k : $\dot{y}_k = 0$
- local tightness and local unemployment rate are directly related:

$$u(\theta_k) \equiv \frac{U_k}{l_k} = \frac{s}{s + f(\theta_k)}$$

MODEL BEVERIDGE CURVE IS AN HYPERBOLA

- balanced flows: $u_k = s / [s + f(\theta_k)]$
- matching function: $f(\theta_k) = \omega \cdot \sqrt{\theta_k} - s$

$$\leadsto u_k = (s/\omega) / \sqrt{v_k/u_k}$$

- Beveridge curve is a rectangular hyperbola, just like in the US:

$$v_k \times u_k = (s/\omega)^2$$

- s/ω : location of the Beveridge curve

BALANCED FLOWS AND RECRUITER-PRODUCER RATIO

- recruiters from household k employed by household j : V_{jk}
 - their services do not deliver direct utility
- producers from household k employed by household j : $c_{jk} = y_{jk} - V_{jk}$
 - their services deliver direct utility
- workers from household k employed by household j :

$$\dot{y}_{jk} = q(\theta_k) \cdot V_{jk} - s \cdot y_{jk} = q(\theta_k) \cdot V_{jk} - s \cdot [c_{jk} + V_{jk}]$$

- flows are balanced in all (j, k) cells: $\dot{y}_{jk} = 0$
- local tightness determines the local recruiter-producer ratio:

$$\tau(\theta_k) \equiv \frac{V_{jk}}{c_{jk}} = \frac{s}{q(\theta_k) - s}$$

PRODUCTIVE EFFICIENCY AT SHOP k

- amount of services consumed:

$$c_k = y_k - V_k = l_k - U_k - V_k = l_k \cdot [1 - u_k - v_k]$$

- maximizing c_k is equivalent to minimizing $u_k + v_k$
- subject to the Beveridge curve $v_k \times u_k = (s/\omega)^2$
- from Michailat, Saez (2023), the solution to the maximization is

$$u_k^* = \sqrt{u_k v_k} = s/\omega, \quad \theta_k^* = 1$$

DIRECTED SEARCH AND PRICE/TIGHTNESS COMPETITION

- all workers from household k charge price p_k per unit time
- expenditure by household j on workers k is

$$p_k \cdot y_{jk} = p_k \cdot [c_{jk} + V_{jk}] = p_k \cdot [1 + \tau(\theta_k)] \cdot c_{jk}$$

- workers are **perfectly substitutable**
 - only $c_j = \int_0^1 c_{jk}(t) dk$ enters the utility function
- $p_k \cdot [1 + \tau(\theta_k)]$ must be the same across sellers (Moen 1997)
 - if not, there are cheaper workers available (lower p_k)
 - or workers that can be hired more easily (lower τ_k)
- there is a price level p so $p_k \cdot [1 + \tau(\theta_k)] = p \cdot [1 + \tau(\theta)]$ for all k

EFFECT OF LOCAL PRICE ON LOCAL TIGHTNESS

- price chosen by household k determines the tightness it faces:

$$\theta_k = \tau^{-1} \left(\frac{p}{p_k} [1 + \tau(\theta)] - 1 \right)$$

- the function τ^{-1} is increasing, so θ_k is decreasing in p_k
- ↪ a high price leads to low tightness, high unemployment
- ↪ a low price leads to high tightness, low unemployment

EFFICIENCY WITHOUT PRICE-ADJUSTMENT COSTS

- seller chooses price to maximize income subject to demand curve
- subject to demand $\theta_k(p_k)$, seller chooses p_k to maximize:

$$p_k \cdot y_k = p \cdot [1 + \tau(\theta)] \cdot \frac{y_k}{1 + \tau(\theta_k)} = p \cdot [1 + \tau(\theta)] \cdot \frac{1 - u(\theta_k)}{1 + \tau(\theta_k)} \cdot l_k$$

- $\tau(\theta), u(\theta), v(\theta)$ are linked by

$$\frac{1 - u(\theta_k)}{1 + \tau(\theta_k)} = 1 - u(\theta_k) - v(\theta_k)$$

- seller sets local tightness θ_k to minimize $u(\theta_k) + v(\theta_k)$
- ⇔ sets unemployment rate u_k to minimize $u_k + v(u_k)$
- ⇔ **unemployment rate u_k is efficient (Moen 1997)**

PRICE RIGIDITY

- unexpected price/wage changes **upset customers/workers**
 - Shiller (1996): higher-than-normal price inflation upsets customers, who feel unfairly treated when they go to the store
 - Bewley (1999): lower-than-normal wages damage workers' morale, who feel unfairly treated
- inflation chosen by household k : $\pi_k = \dot{p}_k/p_k$
- flow disutility when inflation deviates from norm (Rotemberg 1982):

$$\frac{\kappa}{2} \cdot [\pi_k - \pi^*]^2$$

PEOPLE'S PREFERENCES

- household j maximizes utility

$$\int_0^{\infty} e^{-\delta t} \left\{ \ln(c_j(t)) + \sigma \cdot \left[\frac{b_j(t)}{p(t)} - \frac{b(t)}{p(t)} \right] - \frac{\kappa}{2} \cdot [\pi_j - \pi^*]^2 \right\} dt$$

- $\delta > 0$: time discount rate
- $\sigma > 0$: status concerns
- $c_j(t) = \int_0^1 c_{jk}(t) dk$: total consumption of services
- $b_j(t)$: saving in government bonds
- $b(t) = \int_0^1 b_j(t) dj$: aggregate wealth

PEOPLE'S BUDGET CONSTRAINT

- law of motion of government bond holdings for household j :

$$\dot{b}_j = i \cdot b_j - \int_0^1 p_k y_{jk} dk + p_j y_j$$

- because of matching and directed search, expenditure becomes:

$$\begin{aligned} \int_0^1 p_k y_{jk} dk &= \int_0^1 p_k [1 + \tau(\theta_k)] c_{jk} dk \\ &= p \cdot [1 + \tau(\theta)] \cdot \int_0^1 c_{jk} dk \\ &= p \cdot [1 + \tau(\theta)] \cdot c_j \end{aligned}$$

- because of matching and directed search, income becomes:

$$p_j \cdot y_j = p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j$$

SOLUTION OF THE MODEL

SOLVING HOUSEHOLD MAXIMIZATION BY HAMILTONIAN

- Hamiltonian of household j 's maximization is

$$\begin{aligned}\mathcal{H}_j = & \ln(c_j) + \sigma \cdot \left[\frac{b_j}{\rho} - \frac{b}{\rho} \right] - \frac{\kappa}{2} \cdot [\pi_j - \pi^*]^2 \\ & + \mathcal{A}_j \cdot [i \cdot b_j - \rho \cdot [1 + \tau] \cdot c_j + p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j] \\ & + \mathcal{B}_j \cdot \pi_j \cdot p_j.\end{aligned}$$

- control variables: c_j, π_j
- state variables: b_j, p_j
- costate variables: $\mathcal{A}_j, \mathcal{B}_j$
- symmetric solution of model: households behave identically

AGGREGATE SUPPLY: PHILLIPS EQUATION

- from optimal pricing by households:

$$\dot{\pi} = \delta \cdot (\pi - \pi^*) - \frac{1}{\kappa} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u} \right]$$

- κ : price-adjustment cost
- $1 - \frac{u \cdot (1 - u - v)}{v \cdot (1 - 2u)}$: inefficiency of the economy
 - zero $\Leftrightarrow u = v \Leftrightarrow \theta = 1 \Leftrightarrow$ efficiency
 - positive $\Leftrightarrow v > u \Leftrightarrow \theta > 1 \Leftrightarrow$ inefficiently tight
 - negative $\Leftrightarrow u > v \Leftrightarrow \theta < 1 \Leftrightarrow$ inefficiently slack
- in steady state ($\dot{\pi} = 0$), Phillips curve:

$$\kappa \cdot \delta \cdot (\pi - \pi^*) = 1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}$$

AGGREGATE DEMAND: EULER EQUATION

- from optimal consumption and saving by households:

$$\frac{\dot{u}}{1-u} = \delta - [i(\pi) - \pi + \sigma \cdot (1-u) \cdot l]$$

- $i(\pi) - \pi$: real interest rate, financial return on saving
- $\sigma \cdot y$: MRS between wealth & consumption, hedonic return on saving
 - discounted Euler equation (McKay, Nakamura, Steinsson 2017)
- in steady state ($\dot{u} = 0$), Euler curve:

$$\pi = i(\pi) - \delta + \sigma \cdot (1-u) \cdot l$$

DIVINE COINCIDENCE APPEARS IN PHILLIPS CURVE

- Phillips curve is given by

$$\kappa \cdot \delta \cdot (\pi - \pi^*) = 1 - \frac{u}{v} \cdot \frac{1 - u - v}{1 - 2u}$$

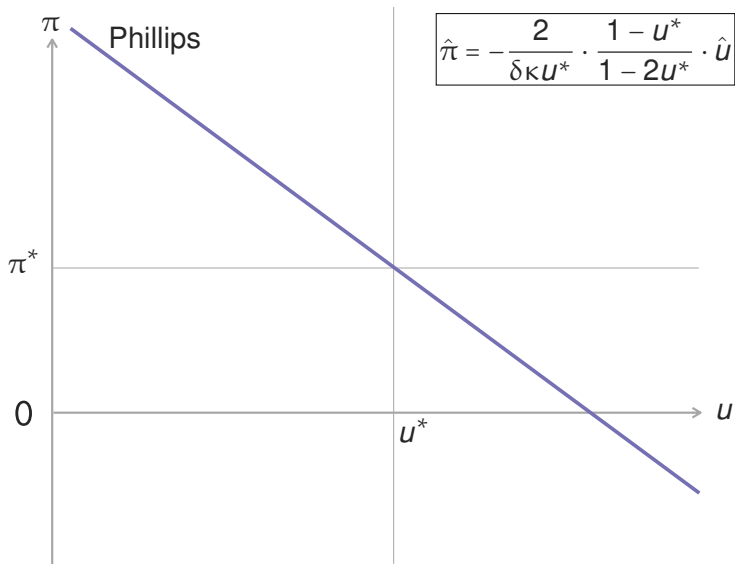
- $\pi = \pi^* \Leftrightarrow u = v \Leftrightarrow \theta = 1 \Leftrightarrow u = u^*$
- Phillips curve goes through (u^*, π^*) so divine coincidence holds
- ~> if monetary policy is set appropriately, inflation is on target whenever unemployment is efficient
- ~> the price and employment mandates are consistent

MONETARY POLICY SATISFYING THE DUAL MANDATE

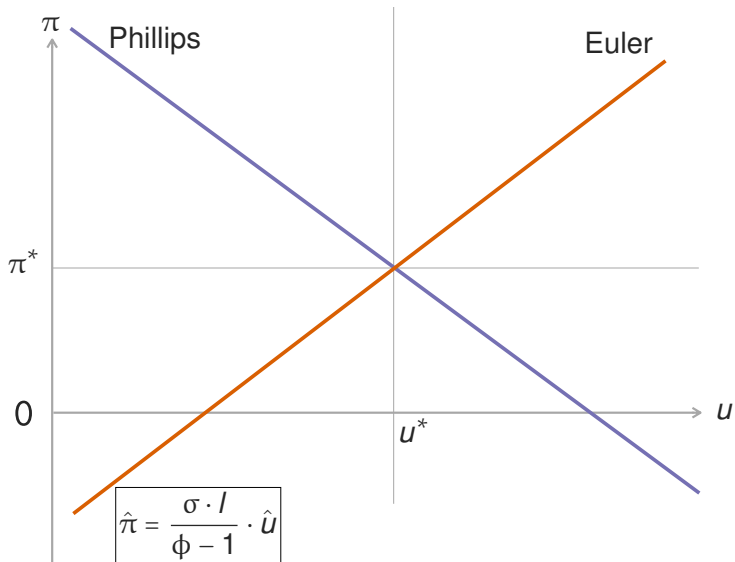
- nominal interest rate i^* ensures:
 - inflation is on target: $\pi = \pi^*$
 - unemployment is efficient: $u = u^*$
- from Euler curve: $i^* = \pi^* + \delta - \sigma \cdot (1 - u^*) \cdot l$
- policy can take different forms:
 - interest-rate peg: $i(\pi) = i^*$
 - Taylor rule with $\phi > 0$: $i(\pi) = i^* + \phi \cdot (\pi - \pi^*)$
- dual-mandate policy also maximizes social welfare

DYNAMICS OF THE MODEL

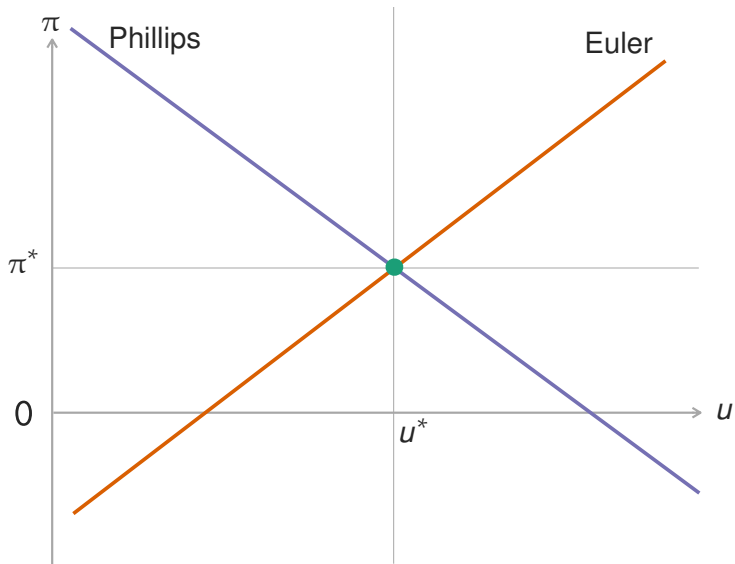
LINEARIZED PHILLIPS CURVE



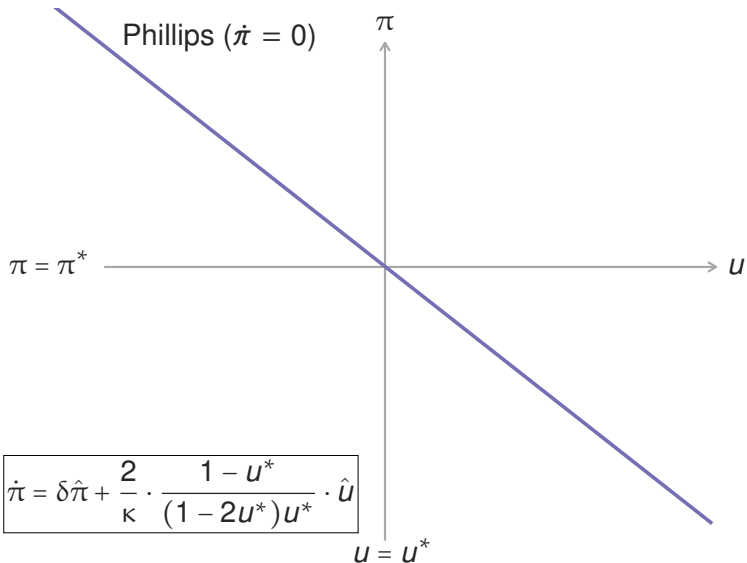
LINEARIZED EULER CURVE



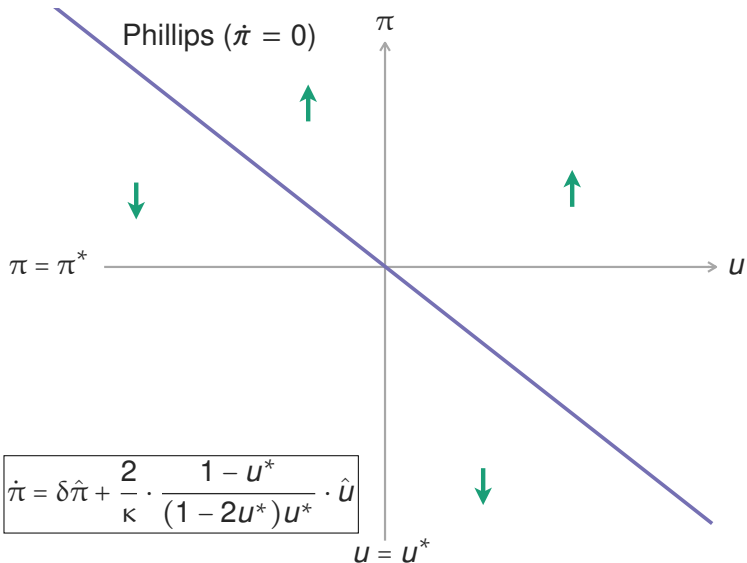
DIVINE COINCIDENCE IN THE EARTHLY MODEL



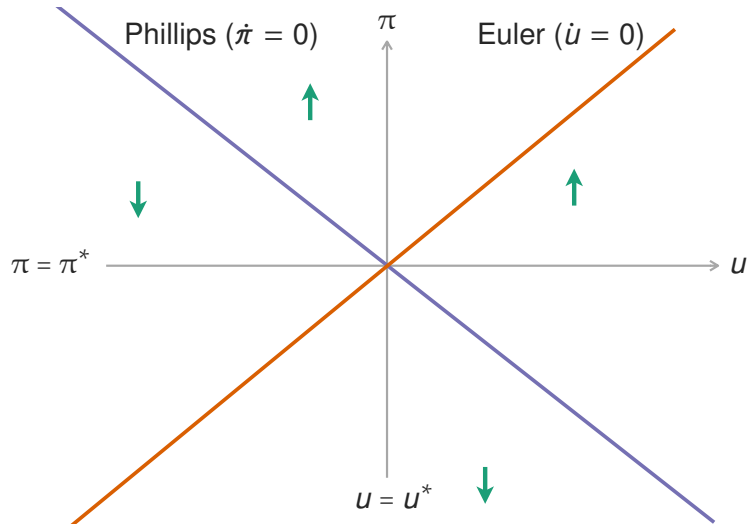
PHASE DIAGRAM OF THE EARTHLY MODEL



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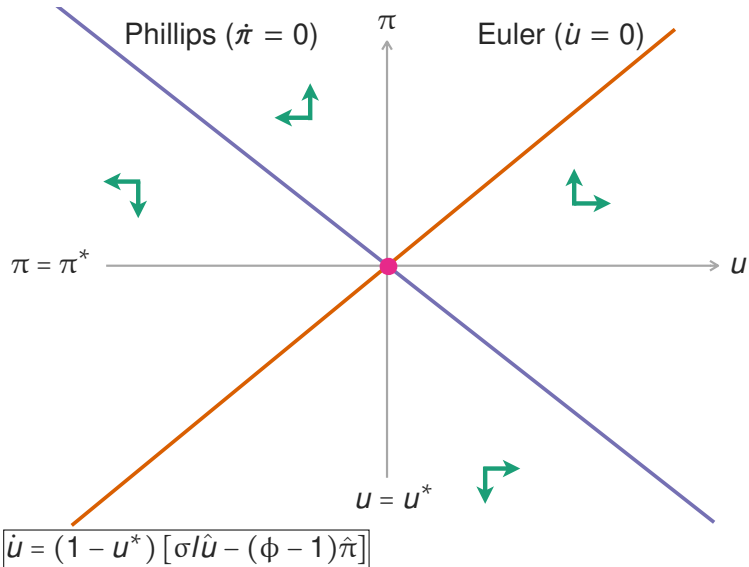


PHASE DIAGRAM OF THE EARTHLY MODEL (TAYLOR RULE)

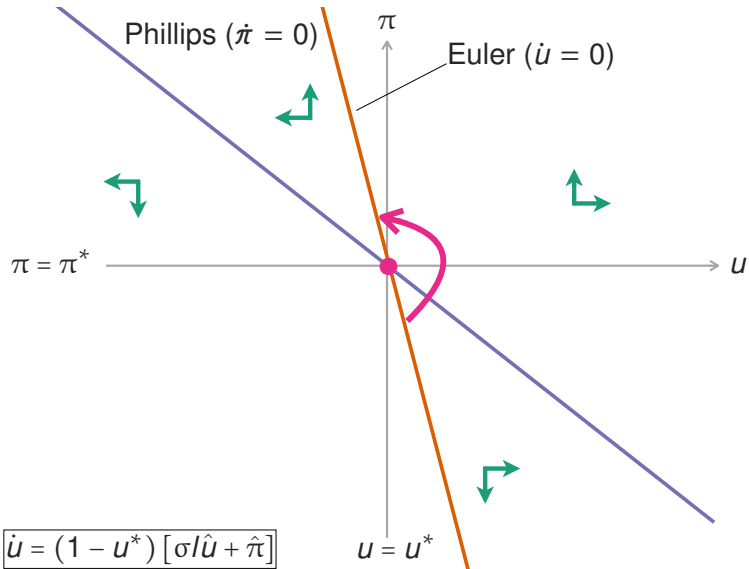


$$\dot{u} = (1 - u^*) [\sigma \hat{u} - (\phi - 1) \hat{\pi}]$$

PHASE DIAGRAM OF THE EARTHLY MODEL (TAYLOR RULE)

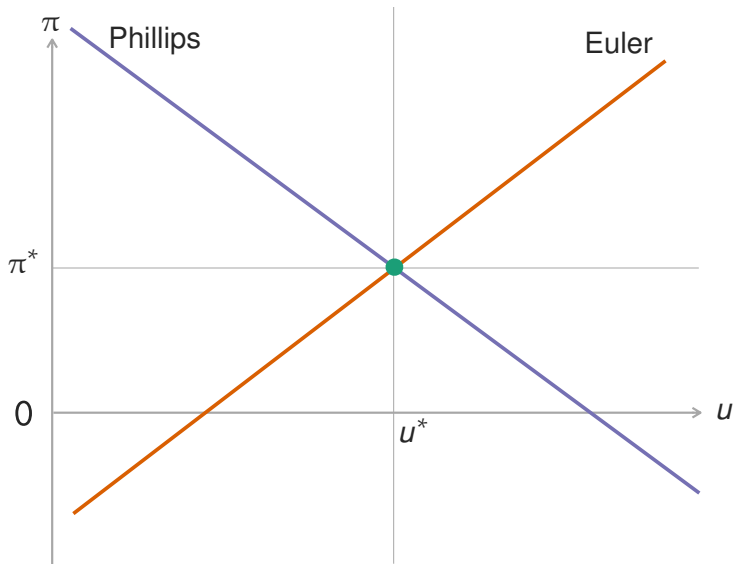


PHASE DIAGRAM OF THE EARTHLY MODEL (PEG)

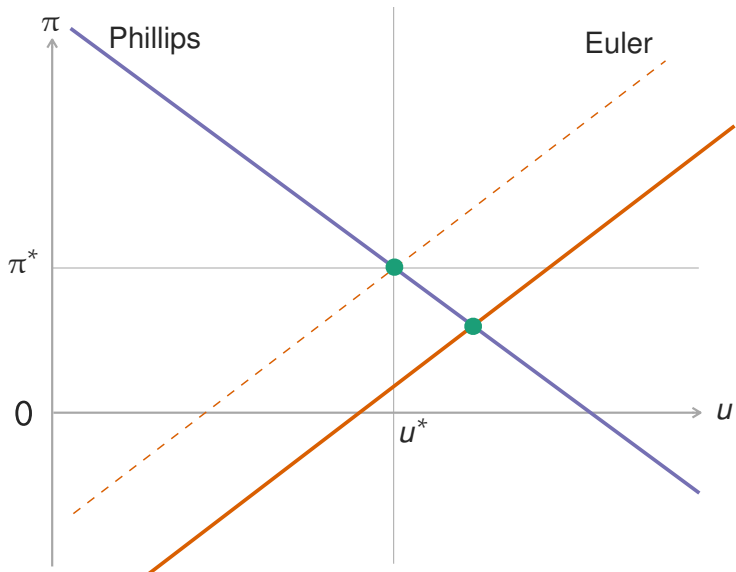


RESPONSE TO DEMAND AND SUPPLY SHOCKS

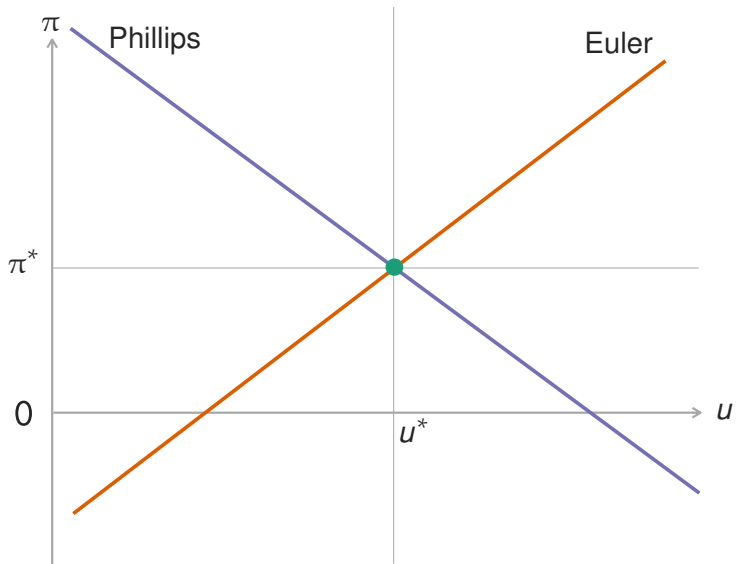
NEGATIVE DEMAND OR MONETARY SHOCK



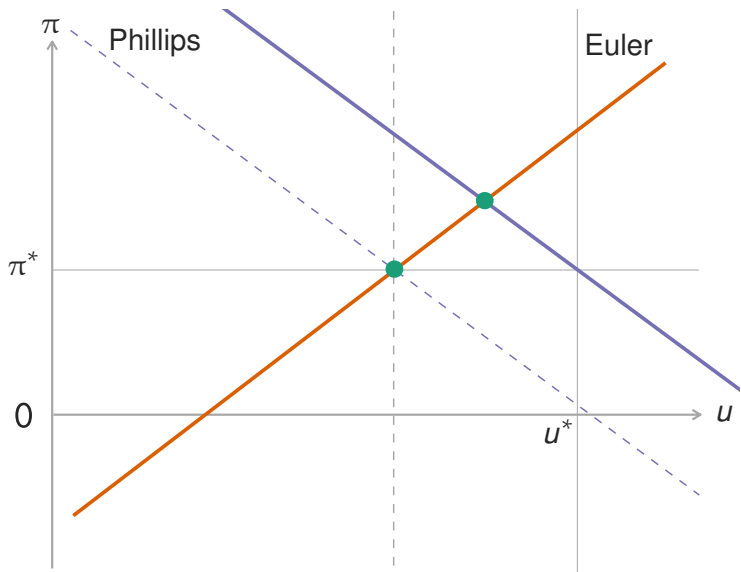
NEGATIVE DEMAND OR MONETARY SHOCK



PANDEMIC SHIFT OF THE BEVERIDGE CURVE

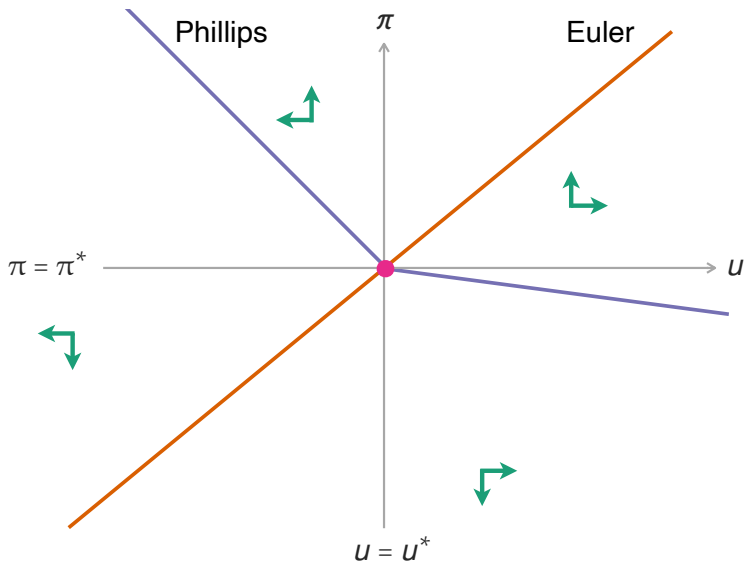


PANDEMIC SHIFT OF THE BEVERIDGE CURVE

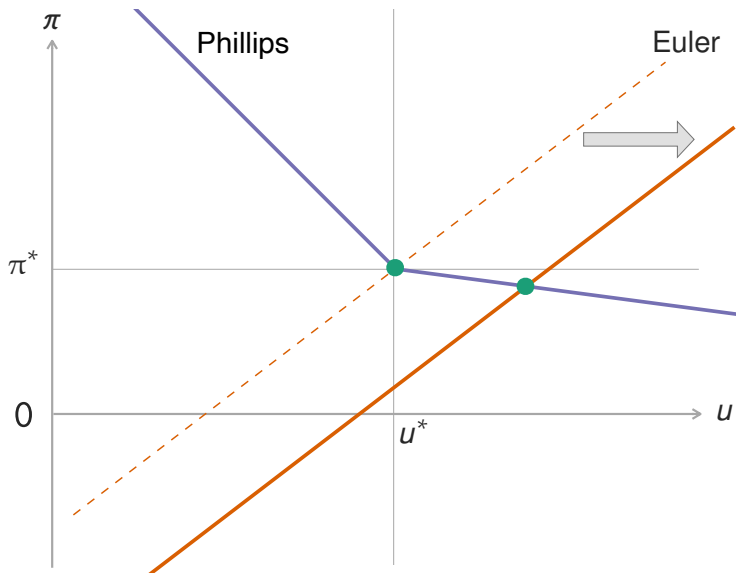


KINK IN THE PHILLIPS CURVE

DOWNWARD WAGE RIGIDITY > UPWARD PRICE RIGIDITY



NEGATIVE DEMAND SHOCK: UNEMPLOYMENT GAP \uparrow



NEGATIVE SUPPLY SHOCK: INFLATION \uparrow

